Formal Geometry

DE

• $\Delta BDE \sim \Delta BAC$, then, by the **AA Similarity Theorem**.

parallel lines \overrightarrow{DE} and \overrightarrow{AC} with transversal \overline{AB} .

In the figure to the right, I see similar triangles: $\Delta BDE \sim \Delta BAC$.

• $\angle B \cong \angle B$ by the reflexive property of congruence.

1) Determine if any similar figures are shown below. Justify your answer by stating the theorem used.

• $\angle BDE \cong \angle BAC$ because they are corresponding angles of

2) Two similar figures of areas of 40 and 90. Find the ratio of their perimeters (small to large.)

Area ratios are the squares of the corresponding linear ratios. Perimeters are linear measures. Therefore, we have the proportion:

 $r^2 = \frac{40}{90}$ (using *r* for ratio)

The small figure is on the top in the fractions and the large figure on the bottom. The perimeter ratio, r, is squared to get the area ratio. Then,

$$r^2 = \frac{40}{90} = \frac{4}{9} \rightarrow r = \sqrt{\frac{4}{9}} = \frac{2}{3}$$
 or 2:3

For # 3 - 4, determine whether each pair of figures is similar. If so write a similarity statement and state the reason.



We only have the sides to work with, so we must check proportions. The easiest way to do this is by increasing the sizes of the sides of the triangles as you move from left to right in the proportions. So, we want to know if:

$$\frac{8}{24} = \frac{9}{27} = \frac{10}{30}$$

Simplifying the fractions, we get: $\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$. Then, **by the SSS Similarity Theorem**, $\Delta FDE \sim \Delta ACB$.

Be careful to name the triangles with corresponding angle names in the same position in the 3-letter triangle names. For example, in the above problem, angles F and A correspond because they are both angles between the largest and smallest sides of their respective triangles.



It looks like we have congruent triangles here, by the SAS congruence theorem. Congruent triangles are similar, so we also have similarity, though in math parlance, similarity is less robust than congruence (i.e., it tells us less about the relationship between the shapes).

In any case, $\Delta FDE \sim \Delta ACB$ by the SAS Similarity Theorem.

Note that I can name the first triangle in the similarity statement using the angle names in any order, but I am then constrained to name the second triangle with corresponding angle names in the same positions in the 3-letter triangle name.

5) If $\triangle EDF \sim \triangle ACB$, then find x.

Let's be careful with letter order in setting up our proportion for this problem.

$$\frac{EF}{ED} = \frac{AB}{AC}$$

$$\frac{x-5}{5} = \frac{13}{x+3}$$

$$(x-5)(x+3) = 13 \cdot 5$$

$$x^{2} - 2x - 15 = 65$$

$$x^{2} - 2x - 80 = 0$$

$$(x-10)(x+8) = 0$$

$$x = 10$$
Notice the



x = 10, -8 Notice that x = -8 would give negative lengths in the diagram, so we discard that solution. So, x = 10.

6) If $\Delta FAT \sim \Delta COW$, then $\frac{FA}{FT} = \frac{?}{?}$.

In constructing proportions for the sides of similar figures, letters must be in the same order on both sides of the equal sign. *FA* are the first two letters of ΔFAT , and *FT* are the first and third letters. We want the same letters of ΔCOW on the right side of the proportion – first two on top of the fraction, first and third on the bottom of the fraction:

$$\frac{FA}{FT} = \frac{CO}{CW}$$

7) Two equilateral triangles are similar. Are they congruent? Explain why or why not.

Sometimes. Similarity requires that corresponding angles be congruent, but not that corresponding sides be congruent. Two equilateral triangles are congruent if their side measures are equal. They are not congruent if their side measures are not equal.

8. Two similar triangles have perimeters of 50 to 30. If the area of the large triangle is 100, find the area of the smaller triangle.

Area ratios are the squares of the corresponding linear ratios. Perimeters are linear measures. Therefore, we have the proportion:

$$\left(\frac{50}{30}\right)^2 = \frac{100}{x}$$

The large triangle is on the top in these fractions and the small triangle on the bottom. The perimeter ratio is squared to get the area ratio. Then,

 $\frac{2500}{900} = \frac{100}{x} \to x = \frac{100 \cdot 900}{2500} = 36 \text{ units}^2$

In problems # 9 - 12, decide whether the statement is True (T) or False (F).

9. All obtuse triangles are ~.

False. Obtuse triangles have exactly one obtuse interior angle, but that obtuse angle can have different measures in two triangles. Therefore, the obtuse angles do not have to be congruent, meaning that the obtuse triangles do not have to be similar.

10. If two \sim triangles have a scale factor of 2: 5, then the ratio of their areas is 4: 25.

True. Area ratios are the squares of the corresponding linear ratios. Scale factors are linear measures. Therefore, we have the proportion:

$$\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}$$

11. If two triangles are similar with ratio of areas of 98:72, then the ratio of their perimeters is 7:6.

True. Area ratios are the squares of the corresponding linear ratios. Perimeters are linear measures. Therefore, we have the proportion:

$$\left(\frac{7}{6}\right)^2 = \frac{7^2}{6^2} = \frac{49}{36} = \frac{98}{72}$$

12. Two isosceles triangles are similar.

False. This is a good one to draw. To the right are two isosceles triangles that are clearly not similar because the angles between the congruent sides differ in the two triangles.

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13. In the diagram shown, a 5-ft tall person casts a shadow of 8 feet when standing 16 feet away from a tree. Find the height of the tree.

$\Delta ABC \cong \Delta DEC.$

Let's label the distances and points. Then, comparing the large triangle to the small triangle, we have:

$$\frac{left \ side \ \Delta ABC}{left \ side \ \Delta DEC} = \frac{bottom \ \Delta ABC}{bottom \ \Delta DEC}$$
$$\frac{AB}{5} = \frac{16+8}{8} \rightarrow AB = \frac{5 \cdot 24}{8} = 15 \text{ feet}$$



14) If the scale factor of two similar polygons is 5:2 and the perimeter of the larger polygon is 45 ft, then find the perimeter of the smaller polygon.

Perimeter ratios are linear ratios. Scale factors are linear also. Therefore, we have the proportion:

$$\frac{5}{2} = \frac{45}{x}$$

 $x = \frac{45 \cdot 2}{5} = 18 \text{ ft}$

For #15 - 17, use the diagram shown.

15. Which triangles are similar, and by what theorem?

In the figure to the right, I see similar triangles: $\triangle AEB \sim \triangle ADC$.

- $\angle A \cong \angle A$ by the reflexive property of congruence.
- $\angle AEB \cong \angle ADC$ because they are corresponding angles of parallel lines \overrightarrow{DC} and \overrightarrow{EB} with transversal \overline{AD} .
- $\Delta AEB \sim \Delta ADC$, then, by the **AA Similarity Theorem**.

16. Find x.

Find x.

$$\frac{left \ side \ \Delta AEB}{left \ side \ \Delta ADC} = \frac{right \ side \ \Delta AEB}{right \ side \ \Delta ADC}$$

$$\frac{6-5}{6} = \frac{2}{x+4+2} \rightarrow \frac{1}{6} = \frac{2}{x+6} \rightarrow x+6 = 12 \rightarrow x = 6$$



17. Find BC and AC.

$$BC = x + 4 = 6 + 4 = 10$$

 $AC = AB + BC = 2 + 10 = 12$

18. Find x and AB.

First, we need to find the similarity, then the appropriate proportion.

• ∠B ≅ ∠B by the reflexive property of congruence.



- $\angle AEB \cong \angle CDB$ because they are corresponding angles of parallel lines \overleftarrow{AE} and \overrightarrow{DC} with transversal \overline{ED} .
- $\Delta AEB \sim \Delta CDB$, then, by the **AA Similarity Theorem**.

The proportion we want must follow the lettering in the similarity.

 $\frac{AB}{CB} = \frac{EB}{DB}$, with the left triangle on the top and the right triangle on the bottom of the proportion.

$$\frac{4x-8}{16} = \frac{5}{4}$$

$$16x - 32 = 80$$

$$16x = 112$$

$$x = 7$$

$$AB = 4x - 8 = 4 \cdot 7 - 8 = 20$$



An angle bisector in a triangle divides the opposite sides into segments that are proportional to the adjacent sides. So,

$$\frac{4}{x-3} = \frac{x-3}{9}$$

(x-3)(x-3) = 4 · 9
x² - 6x + 9 = 36
x² - 6x - 27 = 0
(x-9)(x+3) = 0 \rightarrow x = 9, -3

If x = -3, we have negative side lengths, so we discard the solution x = -3.

If x = 9, the sides of $\triangle BAD$ would be 4, 6, 15, which does not make a triangle (4 + 6 < 15), so we discard the solution x = 9. This problem has no solution.

20) In the figure below, AE // CD and AD intersects CE at B. What is the length of CE?

First, we need to find the similarity, then the appropriate proportion.

- $\angle ABE \cong \angle DBC$ because they are vertical angles.
- $\angle A \cong \angle D$ because they are alternate interior angles of parallel lines \overrightarrow{AE} and \overrightarrow{CD} with transversal \overrightarrow{AD} .
- $\Delta ABE \sim \Delta DBC$, then, by the **AA Similarity Theorem**.

The proportion we want must follow the lettering in the similarity.

 $\frac{AB}{DB} = \frac{EB}{CB}$, with the large triangle on the top and the small triangle on the bottom of the proportion.

$$\frac{10}{5} = \frac{8}{CB}$$

$$10 \cdot CB = 40$$

$$CB = 4$$

$$CE = EB + CB = 8 + 4 = 12$$

21. Find x and y.

The four parallel vertical lines in the diagram divide the horizontal lines into proportional segments.

 $\frac{5}{x} = \frac{4}{3} \rightarrow 4x = 15 \rightarrow x = \frac{15}{4} = 3.75$ $\frac{4+3}{y} = \frac{4}{3} \rightarrow 4y = 21 \rightarrow y = \frac{21}{4} = 5.25$



We have assumed in this problem that y is the distance along the bottom of the two horizontal lines, between the 2nd and 4th vertical lines, as shown in purple in the diagram.

22. Find x.

Given the parallel lines, we can see that $\Delta ACD \sim \Delta ABE$, by the AA Similarity Theorem. Then,

$$\frac{\mathrm{top}}{\mathrm{diagonal}} = \frac{\mathrm{top}}{\mathrm{diagonal}} \rightarrow \frac{8+6}{10+x} = \frac{8}{x}$$

$$14x = 80 + 8x$$

$$6x = 80$$

$$x = \frac{80}{6} = \frac{40}{3} \sim 13.33$$



23. Find y and z.

Given the parallel lines, we can see that $\Delta ACD \sim \Delta ABE$, by the AA Similarity Theorem. Then,

$$\frac{\text{top}}{\text{right}} = \frac{\text{top}}{\text{right}} \rightarrow \frac{8}{y} = \frac{8+6}{20}$$

$$14y = 160$$

$$y = \frac{160}{14} = \frac{80}{7} \sim 11.43$$



Given the parallel lines, $\angle ABE$ and $\angle ACD$ are corresponding angles, so they are congruent. Then,

$$z = 30.$$

24. Two similar rectangles have a scale factor of 9:8. Find the ratio of their perimeters.

Perimeter ratios are linear ratios. Scale factors are linear also. So, the ratios are the same.

Perimeter ratio = $\frac{9}{8}$ or 9:8

For # 25 – 26: Apply the dilation $D: (x, y) \rightarrow (2x, 2y)$ to the triangle given below.

25) Find the coordinates of the triangle after the dilation.

The coordinates of the preimage are (2, 1), (4, 1), (4, -3).

The dilation doubles all *x*- and *y*-values. So, the coordinates of the image are:

(4, 2), (8, 2), (8, -6)

26) Find the perimeter of the triangle after the dilation. (Round answer to 3 decimal places)

From (4, 2) to (8, 2), the distance is: 8 - 4 = 4. From (8, 2) to (8, -6), the distance is: 2 - (-6) = 8. From (8, -6) to (4, 2), the distance is: $\sqrt{(4 - 8)^2 + (2 - (-6))^2} = \sqrt{80} \sim 8.944$ Perimeter $\sim 4 + 8 + 8.944 = 20.944$





For # 27 - 28, solve for x. (Round answers to 3 decimal places if necessary.)

29. Given: $\overline{AC} \parallel \overline{DE}$ Prove: $\triangle ABC \sim \triangle DBE$

Step	Statement	Reason
1	$\overline{AC} \parallel \overline{DE}$	Given.
2	$\angle B \cong \angle B$	Reflexive property of congruence.
3	$\angle CAB \cong \angle EDB$	If parallel lines are cut by a transversal, then corresponding angles are congruent.
4	$\Delta ABC \cong \Delta DBE$	AA Similarity Theorem. Angles in Steps 2 and 3.



30. Given: BD || CE.

Prove: $\frac{AB}{AC} = \frac{AD}{AE}$

A	
В	~
°	E

Step	Statement	Reason
1	$\overline{BD} \parallel \overline{CE}$	Given.
2	$\angle A \cong \angle A$	Reflexive property of congruence.
3	$\angle ABD \cong \angle ACE$	If parallel lines are cut by a transversal, then corresponding angles are congruent.
4	$\Delta ABD \cong \Delta ACE$	AA Similarity Theorem. Angles in Steps 2 and 3.
5	$\frac{AB}{AC} = \frac{AD}{AE}$	Corresponding sides in similar triangles are proportional.

31. **Given:** $\angle X \cong \angle ZBA$.

Prove: $AZ \cdot XY = AB \cdot ZY$



Step	Statement	Reason
1	$\angle X \cong \angle ZBA$	Given.
2	$\angle Z \cong \angle Z$	Reflexive property of congruence.
3	$\Delta ZXY \cong \Delta ZBA$	AA Similarity Theorem. Angles in Steps 1 and 2.
4	$\frac{AZ}{ZY} = \frac{AB}{XY}$	Corresponding sides in similar triangles are proportional.
5	$AZ \cdot XY = AB \cdot ZY$	Multiplicative property of equality (applied twice).

32. C, B, and D are midpoints. Find the perimeter of $\triangle AEF$ if BD = 9.2, DF = 6, and CD = 6.5.

The four small triangles formed by connecting midpoints in the large triangle are all congruent. Further, the perimeter of the outer triangle will be double the perimeter of any of the four interior triangles. (Also, the area of the outer triangle will be four times the area of any



of the four interior triangles. We don't need to know that for this problem.)

We are given the three lengths shown in magenta in the diagram. Let's use the perimeter of ΔDBF as our basis to calculate the perimeter of ΔAEF .

 $P(\Delta DBF) = BD + BF + DF$

Of the three distances in the formula, we are missing BF, but fortunately we know that BF = CD = 6.5. Then,

$$P(\Delta DBF) = BD + BF + DF = 9.2 + 6.5 + 6 = 21.7.$$

$$P(\Delta AEF) = 2 \cdot P(\Delta DBF) = 2 \cdot 21.7 = 43.4$$
 units

33) A 6 foot tall boy is standing 12 feet from a 20 foot tall lamppost. How long is his shadow, if it has a common endpoint with the shadow of the lamppost? (Hint: draw a picture)

Notice that my lamppost is cleverly disguised as a tree. It's part of the community deciding to go green.

$$\Delta ABC \cong \Delta DEC.$$

Let the length of the shadow be x.

Comparing the large triangle to the small triangle, we have:

$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{20}{6} = \frac{12 + x}{x}$$

$$20x = 72 + 6x$$

$$14x = 72$$

$$x = \frac{72}{14} = \frac{36}{7}$$
 feet ~ 5. 14 feet



For #34-35: Factor Completely:

34) $-5x^3 - 45x$ $-5x^3 - 45x$ $-5x(x^2 + 9)$ 35) $4x^4 - 64x^2$ $4x^4 - 64x^2$ $4x^2(x^2 - 16)$ $4x^2(x - 4)(x + 4)$



36) Given: $\triangle ADE \sim \triangle ABC$

Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Step	Statement	Reason
1	$\Delta ADE \cong \Delta ABC$	Given.
2	$\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$	Corresponding sides in similar triangles are proportional.
3	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	Simplification.
4	$\frac{DB}{AD} = \frac{EC}{AE}$	Subtraction property of equality.
5	$\frac{AD}{DB} = \frac{AE}{EC}$	Multiplicative inverses of equal non-zero fractions are equal.